Questions and answers for Module 3

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1 Questions

1. What is the normalization condition for a two particle Hilbert space ?
2. Write the discrete wave functions for the particle in a box.
3. Write the expression for the discrete energy levels for a quantum harmonic oscillator.
4. Write the properties for P-space and Q-space.
5. Write the evolution equation for the density operator.

2 Answers

1. $$\langle x_1' x_2' | x_1 x_2 \rangle = \delta(x_1 - x_1')\delta(x_2 - x_2').$$

2. $$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right),$$

   when n is even and

   $$\psi_n(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi x}{L}\right),$$

   when n is odd.

3. $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$.

4. $$P = P^\dagger \quad \text{and} \quad P^2 = P.$$

   Let Q be the complementary projection operator of P which is defined as

   $$Q \equiv I - P.$$

   Q also satisfies the following relations,

   $$Q = Q^\dagger \quad \text{and} \quad Q^2 = Q.$$

   As Q always projects a state that is orthogonal to $|\psi_0\rangle$ in the manifold of the Hilbert space, one can write

   $$\begin{aligned} Q|\psi\rangle &= (I - P)|\psi\rangle, \\ &= |\psi\rangle - P|\psi\rangle, \\ &= |\psi\rangle - P(|\psi_0\rangle + |\phi\rangle), \\ &= |\psi\rangle - P|\psi_0\rangle - P|\phi\rangle. \end{aligned}$$

5. $$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H, \rho].$$